**ME EN 541 – HW 3**

Handed out 6 Feb 2024

Due 12 Feb 2024

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| **Problem** | **Points** | **Topic** |
| **1** | 15 | Richardson extrapolation |
| **2** | 20 | Models with source terms |
| **3** | 20 | Models with source terms |

1. Explore Richardson extrapolation on HW 2 Problem 5 as follows.
   1. Calculate the left-most (“base”) temperature simulated using 5, 10, and 20 control volumes. Estimate the order of the simulation using these values.
   2. Predict the grid-independent base temperature using the estimated simulation order from part (a) and the base temperature values from the 10- and 20-control volume cases.
   3. Plot base temperature for 5, 10, 20, 40, …, 5120, 10240 control volumes. Show the predicted grid-independent solution from part b on this plot. Also generate a table that lists, for these control volumes, the number of control volumes, the simulated base temperature, and the percent difference between the simulated solutions and the grid-independent solution.

**Problem Setup for Problems 2 and 3**

Consider cooling of a circular fin by means of convective and radiative heat transfer along its length. For constant diameter, the steady state conservation equation is



where *k* is the thermal conductivity, *D* the diameter, *T*∞ the ambient temperature, *Tsurr* the temperature of the surroundings, ** the fin surface emissivity, and ** the Stefan-Boltzmann constant (5.67×10−8 W/m2K4).

Formulate this problem for solution via the control-volume method, and apply the two cases specified below. For both cases, the temperature at the base is fixed, the tip of the fin can be considered to be adiabatic, and the following parameters apply:

*L* = 2 cm, *D* = 3 mm

*k* = 401 W/mk (copper), *h* = 10 W/m2K

*TB* = 400 K, *T∞* = 273 K, *Tsurr* = 273 K

Note the following in developing your model:

* An iterative method is required since the source term is nonlinear. **Use the TDMA method** to solve the linear system at each iteration.
* Iterate until the maximum temperature change at any node between iterations is < 0.0001.
* Appropriately linearize the source term.

2. Validate your model by predicting the temperature distribution *T*(*x*) and the fin heat transfer *qf* for the convection-only case (** = 0). For this case there is an analytical solution:



where *n*2 = 4*h*/*Dk*. Plot the maximum error vs. number of control volumes.

3. Predict and plot the steady-state temperature distribution for the combined convection and radiation case with *h* = 10 W/m2K and ** = 1. Use an appropriate number of CVs as determined from the previous problem. Comparing this problem with the previous problem, comment on the relative importance of convective vs. radiative heat transfer on the tip temperature.